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Proactively Accountable  
Anonymous Messaging in Verdict

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YALEU/DCS/TR-1478  
June 2013

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## *Extended Version*

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### Abstract

Among anonymity systems, DC-nets have long held attraction for their resistance to traffic analysis attacks, but practical implementations remain vulnerable to internal disruption or “jamming” attacks, which require time-consuming detection procedures to resolve. We present Verdict, the first practical anonymous group communication system built using *proactively verifiable* DC-nets: participants use public-key cryptography to construct DC-net ciphertexts, and use zero-knowledge proofs of knowledge to detect and exclude misbehavior *before* disruption. We compare three alternative constructions for verifiable DC-nets: one using bilinear maps and two based on simpler ElGamal encryption. While verifiable DC-nets incur higher computational overheads due to the public-key cryptography involved, our experiments suggest that Verdict is practical for anonymous group messaging or microblogging applications, supporting groups of 100 clients at 1 second per round or 1000 clients at 10 seconds per round. Furthermore, we show how existing symmetric-key DC-nets can “fall back” to a verifiable DC-net to quickly identify misbehavior, speeding up previous detections schemes by two orders of magnitude.

### 1 Introduction

A right to anonymity is fundamental to democratic culture, freedom of speech [3, 48], peaceful resistance to repression [41], and protecting minority rights [47]. Anonymizing relay tools, such as Tor [19], offer practical and scalable anonymous communication but are vulnerable to traffic analysis attacks [4, 36, 40] feasible for powerful adversaries, such as ISPs in authoritarian states.

*Dining cryptographers networks* [13] (*DC-nets*) promise security even against traffic analysis attacks, and recent systems such as Herbivore [26, 46] and Dissent [15, 54] have improved the scalability of DC-net-style systems. However, these systems are still vulnerable to internal *disruption* attacks in which a misbehaving member anonymously “jams” communication, either

completely or selectively. Dissent includes a *retrospective blame* procedure that can eventually exclude disruptors, but at high cost: tracing a disruptor in a 1,000-member group takes over 60 minutes [54], and the protocol makes no communication progress until it restarts “from scratch.” An adversary who infiltrates such a group with  $f$  colluding members can “sacrifice” them one at a time to disrupt *all* communication for  $f$  contiguous hours at any time—long enough time to cause a communications blackout before or during an important mass protest, for example.

Verdict, a novel but practical group anonymity system, thwarts such disruptions while maintaining DC-nets’ resistance to traffic analysis. At Verdict’s core lies a *verifiable* DC-net primitive, derived from theoretical work proposed and formalized by Golle and Juels [27], which requires participating nodes to prove *proactively* the well-formedness of messages they send. The first working system we are aware of to implement verifiable DC-nets, Verdict supports three alternative schemes for comparison: a pairing scheme using bilinear maps similar to the Golle-Juels approach, and two schemes based on ElGamal encryption in conventional integer or elliptic curve groups. Verdict incorporates this verifiable core into a client/server architecture like Dissent’s [54], to achieve scalability and robustness to client churn. As in Dissent, Verdict maintains security as long as *at least one* of the participating servers is honest, and participants need not know or guess which servers are honest.

Due to their reliance on public-key cryptography, verifiable DC-nets incur higher computation overheads than traditional DC-nets, which primarily use symmetric-key cryptography (e.g., AES). We expect this CPU cost to be acceptable in applications where messages are usually short (e.g., chat or microblogging), where costs are dominated by network delays, or in groups with relatively open or antagonistic membership where disruption risks may be high. Under realistic conditions, we find that Verdict can support groups of 100 users while maintain-

ing 1-second messaging latencies, or 1000-user groups with 10-second latencies. In a trace-driven evaluation of full-system performance for a microblogging application, Verdict is able to keep up with symmetric-key DC-nets in groups of up to 250 active users.

In contrast with the above “purist” approach, which uses expensive public-key cryptography to construct *all* DC-net ciphertexts, Verdict also implements and evaluates a *hybrid* approach that uses symmetric-key DC-nets for data communication when not under disruption attack, but leverages verifiable DC-nets to enable the system to respond much more quickly and inexpensively to disruption attacks. Dissent uses a *verifiable shuffle* [38] to broadcast an *accusation* anonymously; this shuffle dominates the cost of identifying disruptors. By replacing this verifiable shuffle with a verifiable DC-nets round, Verdict preserves the disruption-free performance of symmetric-key DC-nets, but reduces the time to identify a disruptor in a 1000-node group by two orders of magnitude, from 20 minutes to 26 seconds.

This paper’s primary contributions are:

- the first working implementation and experimental evaluation of verifiable DC-nets in a practical anonymous communication system,
- two novel verifiable DC-nets constructions using standard modular integer and elliptic curve groups, offering an order of magnitude lower computational cost than the original pairing approach [27],
- a hybrid system design that preserves performance of symmetric-key DC-nets, while reducing disruption resolution costs by two orders of magnitude, and
- experimental evidence suggesting that verifiable DC-nets may be practical for realistic applications, such as anonymous microblogging.

Section 2 introduces DC-nets and the disruption problem. Section 3 outlines Verdict’s architecture and adversary model, and Sections 4 and 5 describe its messaging protocol and cryptographic schemes. Section 6 presents application scenarios and evaluation results, Section 7 describes related work, and Section 8 concludes.

## 2 Background and Motivation

This section first introduces the basic DC-nets concept and known generalizations, then motivates the need for proactive accountability.

### 2.1 Anonymity with Strong Adversaries

To make the need for traffic-analysis-resistant anonymity systems more concrete, consider a political journalist who obtains some important secret government documents (e.g., the *Pentagon Papers*) from a confidential source. If the journalist publishes these documents under her own name, the journalist might risk prosecution

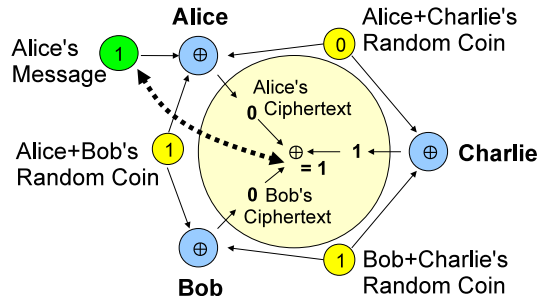


Figure 1: The basic DC-nets algorithm

or interrogation, and she might be pressured to reveal the source of the documents.

To reduce such risks, a number of political journalists could form a Verdict communication group. Any participating journalist may then *anonymously broadcast* the documents to the entire group of journalists, such that no member of the group can determine which journalist sent the documents. With Verdict, even if a government agency plants agents within the group of journalists and observes *all network traffic* during a protocol run, the agency remains unable to learn the source of the leak.

Existing systems such as Tor, which are practical and scalable but vulnerable to known traffic analysis attacks [17, 19, 34], cannot guarantee security in this context. For example, if a US journalist posts a leak to a US website, via a Tor connection whose entry and exit relays are in Europe, then an eavesdropper capable of monitoring transatlantic links [33] can de-anonymize the user via traffic analysis [19, 37]. Prior anonymity systems attempting to offer resistance to traffic analysis, discussed in Section 7, suffer from poor performance or vulnerability to active denial-of-service attacks.

### 2.2 DC-nets Overview

DC-nets [13] provide anonymous broadcast within a *group* of participants, who communicate lock-step in a series of *rounds*. In a given round, each group member contributes an equal length ciphertext that, when combined with all other members’ ciphertexts, reveals one or more cleartext messages. All group members know that each message was sent by *some* group member—but do not know *which* member sent each message.

In its simplest form, illustrated in Figure 1, we assume one group member wishes to broadcast a 1-bit message anonymously. To do so, every pair of members flips a coin, secretly agreeing on the random outcome of that coin flip. An  $N$ -member group thus flips  $N(N - 1)/2$  coins in total, of which each member observes the outcome of  $N - 1$  coins. Each member then XORs together the values of the  $N - 1$  coins she observes, additionally the member who wishes to broadcast the 1-bit

message XORs in the value of that message, to produce that member’s DC-nets *ciphertext*. Each group member then broadcasts her 1-bit ciphertext to the other members. Finally, each member collects and XORs all  $N$  members’ ciphertexts together. Since the value of each shared coin is XORed into exactly two members’ ciphertexts, all the coins cancel out, leaving only the anonymous message, while provably revealing no information about which group member sent the message.

## 2.3 Practical Generalizations

As a standard generalization of DC-nets to communicate  $L$ -bit messages, all members in principle simply run  $L$  instances of the protocol in parallel. Each pair of members flips and agrees upon  $L$  shared coins, and each member XORs together the  $L$ -bit strings she observes with her optional  $L$ -bit anonymous message to produce  $L$ -bit ciphertexts, which XOR together to reveal the  $L$ -bit message. For efficiency, in practice each pair of group members forms a cryptographic shared secret—via Diffie-Hellman key agreement, for example—then group members use a cryptographic pseudo-random number generator (PRNG) to produce the  $L$ -bit strings.

As a complementary generalization, we can use any finite alphabet or group in place of coins or bits, as long as we have: (a) a suitable combining operator analogous to XOR, (b) a way to encode messages in the chosen alphabet, and (c) a way to generate complementary pairs of one-time pads in the alphabet that cancel under the chosen combining operator. For example, the alphabet might be 8-bit bytes, the combining operator might be addition modulo 256, and from each pairwise shared secret, one member of the pair generates bytes  $B_1, \dots, B_k$  from a PRNG, while the other member generates corresponding two’s complement bytes  $-B_1, \dots, -B_k$ .

## 2.4 Disruption and Verifiable DC-nets

A key weakness of DC-nets is that a single malicious insider can easily block all communication. An attacker who transmits arbitrary bits—instead of the XORed ciphertext that the protocol prescribes—unilaterally and *anonymously* jams all DC-net communication.

In many online venues such as blogs, chat rooms, and social networks, some users may have legitimate needs for strong anonymity—protest organizers residing in an authoritarian state, for example—while other antagonistic users (e.g., secret police infiltrators) may attempt to block communication if they cannot de-anonymize “unapproved” senders. Even in a system like Dissent that can *eventually* trace and exclude disruptors, an adversary with multiple colluding dishonest group members may still be able to slow or halt communication for long enough to ruin the service’s usability for honest participants. Further, if the group’s membership is open enough

to allow new disruptive members to join more quickly than the tracing process operates, then these infiltrators may be able to shut down communication permanently.

Verifiable DC-nets [27] leverage algebraic groups, such as elliptic curve groups, as the DC-nets alphabet. Using such groups allows for disruption resistance, by enabling members to *prove* the correctness of their ciphertexts’ construction without compromising the secrecy of the shared pseudo-random seeds. Using a hybrid approach that combines a traditional DC-net with a verifiable DC-net, Verdict can achieve the messaging latency of a traditional XOR-based DC-net while providing the strong disruption-resistance of verifiable DC-nets.

## 3 Verdict Architecture Overview

In this section, we describe the individual components of Verdict and how they combine to form the overall anonymous communication system.

### 3.1 Deployment and Adversary Model

Verdict builds on Dissent [53, 54] and uses the *multi-provider cloud* model illustrated in Figure 2 (a) to achieve scalability and resilience to ordinary node and link failures. In this model, a communication group consists of mostly unreliable *clients*, and a few *servers* we assume to be highly available and well-provisioned. Servers in a group should be administered independently—each furnished by a different *anonymity provider*, for example—to limit risk of all servers being compromised and colluding against the clients. The servers may be geographically or topologically close, however—possibly even hosted in the same data center, in locked cages physically and administratively accessible only to separate, independent authorities.

Clients directly communicate, at a minimum, with a single upstream server, while each server communicates with all other servers. This topology, shown in Figure 2 (b), reduces the communication and computation burden on the clients, and enables the system to make progress regardless of client churn. In particular, clients need not know which other clients are online at the time they submit their DC-net ciphertexts to their upstream server; clients only assume that all *servers* are online.

To ensure anonymity, clients need *not* assume that any particular server is trustworthy—a client need not even trust its immediately upstream server. Instead, **clients trust only that there exists at least one one honest server**, an assumption previously dubbed *anytrust* [53, 54], as a trust analog to anycast communication.

Verdict, like Dissent, achieves security under the anytrust assumption through the DC-nets key-sharing model shown in Figure 2 (c). Each client shares a secret with *every* server, rendering client ciphertexts indecipherable without the cooperation of *all* servers, and

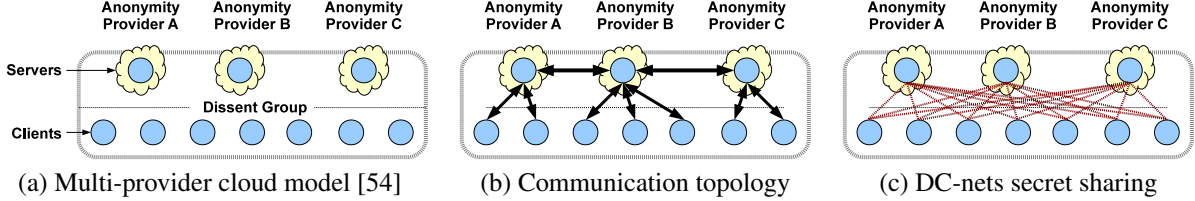


Figure 2: Verdict/Dissent deployment model, physical communication topology, and DC-nets secret sharing

hence protecting a client’s anonymity even if its immediately upstream server is malicious. Each client ultimately obtains an anonymity set consisting of the set of all honest clients, provided that the anytrust assumption holds, and provided the message contents themselves do not in some way reveal the sender’s identity.

A malicious server might refuse to service honest clients, but such refusal does not compromise clients’ anonymity—victims can simply switch to a different server. Although not yet supported in our Verdict prototype, Section 4.6 discusses how one might use threshold secret sharing to tolerate server failures, at the cost of requiring that we assume multiple servers are honest.

### 3.2 Security Goals

Verdict’s goal is to offer anonymity and disruption resistance in the face of a strong adversary who can potentially monitor all network links, modify packets as they traverse the network, and compromise a potentially large fraction of a group’s participating members. We say that a participant is *honest* if it follows the protocol exactly and does not collude with or leak secret information to other nodes. A participant is *dishonest* otherwise. Dishonest nodes can exhibit *Byzantine behavior*—they can be arbitrarily incorrect and can even just “go silent.”

The system is designed to provide anonymity among the set of *honest* participants, who remain online and uncompromised throughout an interaction period, and who do not compromise their identity via the content of the messages they send. We define this set of honest and online participants as the *anonymity set* for a protocol run. If a group contains many colluding dishonest participants, Verdict can anonymize the honest participants only among the remaining subset of *honest* members: in the worst case of a group containing only one honest member, for example, Verdict operates but can offer that member no meaningful anonymity.

Similarly, Verdict does not prevent long-term intersection attacks [30] against otherwise-honest participants who repeatedly come and go during an interaction period, leaking information to an adversary who can correlate online status with linkable anonymous posts. Reasoning about anonymity sets generally requires making inherently untestable assumptions about *how many* group

members may be dishonest or unreliable, but Verdict at least does not assume that the honest participants know *which* other participants are honest and reliable.

Finally, Verdict’s disruption-resistant design addresses *internal* disruption attacks by misbehaving anonymous participants, a problem specific to anonymous communication tools and particularly DC-nets. Like any distributed system, Verdict may be vulnerable to more general network-level Denial-of-Service (DoS) attacks as well, particularly against the servers that are critical to the system’s availability and performance. Such attacks are important in practice, but not specific to anonymous communication systems. This paper thus does not address general DoS attacks since well-known defenses apply, such as server provisioning, selective traffic blocking, and proof-of-life or proof-of-work challenges.

## 4 Protocol Design

Verdict consists of two major components: the messaging protocol, and the cryptographic primitive clients and servers use to construct their ciphertexts. This section describes the Verdict messaging protocols, and the following section describes the cryptographic constructions.

### 4.1 Core Verdict Protocol

Figure 3 summarizes the steps comprising a normal-case run of the Verdict protocol. This protocol represents a direct adaptation of the DC-nets scheme (Section 2.2) to the two-level communication topology illustrated in Figure 2 (b), and to the client/server secret-sharing graph in Figure 2 (c). As in Dissent, Verdict’s anonymity guarantee relies on Chaum’s original security analysis [13], in which an honest node’s anonymity set consists of the set of honest nodes that remain connected in the secret-sharing graph after removing links to dishonest nodes. Since each client shares a secret with every server, and we assume that there exists at least one honest server, this honest server forms a “hub” connecting all honest nodes. This anonymity property holds regardless of physical communication topology, which is why the clients need not trust their immediately upstream server.

The ciphertext- and proof-generation processes assume that communication in the DC-net is broken up into *time slots* (akin to TDMA), such that only one client—

1. **Client Ciphertext Generation.** Each client  $i$  generates a client ciphertext, and submits this ciphertext to client  $i$ 's upstream server. If client  $i$  is the anonymous owner of the current slot, the client computes and submits a slot owner ciphertext using her pseudonym secret key and her plaintext message  $m$ .
  2. **Client Set Sharing.** After receiving valid client ciphertexts from its currently connected downstream clients, each server  $j$  broadcasts to all servers its set  $C_j$  of collected client ciphertexts.
  3. **Server Ciphertext Generation.** After receiving client ciphertext sets from all servers, each server  $j$  computes  $C = \bigcup_k C_k$ , the set of client ciphertexts collected by *all* servers. Server  $j$  then uses  $C$  to compute a server ciphertext corresponding to the set of submitted client ciphertexts. Server  $j$  broadcasts this server ciphertext to all other servers.
  4. **Plaintext Reveal.** After receiving a server ciphertext from every other server, each server  $j$  combines the  $|C|$  client ciphertexts and  $M$  server ciphertexts to reveal the plaintext message  $m$ . Server  $j$  signs  $m$  and broadcasts its signature  $\sigma_j$  to all servers.
  5. **Plaintext Sharing.** After receiving valid signatures from all servers, server  $j$  sends the plaintext message  $m$  and the  $M$  signatures  $\sigma_1, \dots, \sigma_M$  (one from each server) to its downstream clients.
  6. **Client Verification.** Upon receiving the plaintext  $m$  and  $M$  valid signatures from its upstream server, client  $i$  accepts the plaintext message and considers the messaging round to have completed successfully.
- All messages sent over the network include a session nonce and are signed with the sender's long-term well-known (non-anonymous) signing key.*

Figure 3: Core Verdict messaging protocol

the slot's *owner*—is allowed to send an anonymous message in each time slot. The owner of a slot is the client who holds the private key corresponding to a *pseudonym public key* assigned to the slot. To maintain the slot owner's anonymity, *no one must know* which client owns which transmission slot. Section 4.3 below describes the assignment of pseudonym keys to transmission slots.

Figure 4 shows an example DC-net transmission schedule with three slots, owned by pseudonyms A, B, and C. Each slot owner can transmit one message per *messaging round*, and the slot ordering in the schedule remains the same for the duration of the Verdict *session*.

## 4.2 Verifiable Ciphertexts in Verdict

While Verdict's anonymity derives from the same principles as Dissent's, the key difference is in the “alpha-

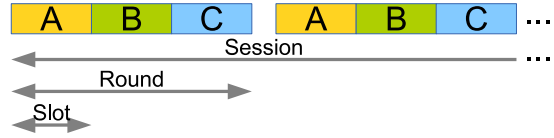


Figure 4: Example DC-net transmission schedule, where anonymous authors A, B, and C transmit in each round.

bet” with which Verdict generates DC-net ciphertexts, and in the way Verdict combines and checks those ciphertexts. Dissent uses a symmetric-key cryptographic pseudo-random number generators (PRNG) to generate shared secrets, and uses bitwise XOR to combine them and later to reveal the plaintext message. While efficient, the symmetric-key approach offers no way to check that any node's ciphertext was generated correctly until the final cleartext messages are revealed. If any node corrupts a protocol round by sending an incorrect ciphertext, Dissent can eventually identify that node only via a complex retroactive *blame* procedure.

Verdict, in contrast, divides messages into chunks small enough to be encoded into elements of algebraic groups, such as Schnorr [44] or elliptic curve groups, to which known proof-of-knowledge techniques apply. Section 5 later outlines three particular ciphertext generation schemes that Verdict implements, although Verdict's architecture and protocol design is agnostic to the specific scheme. These schemes may be considered analogous to “pluggable” ciphersuites in SSL/TLS.

Thus, any Verdict ciphertext is generated *on behalf* of the holder of some particular pseudonym keypair. While the details of a ciphertext correctness proof depend on the particular scheme, all such proofs have the general form of an “either/or” knowledge proof, of the type systematized by Camenisch and Stadler [11]. In particular, a ciphertext correctness proof attests that either:

- the ciphertext is an encryption of *any* message, and the producer of the ciphertext holds the *private* part of the pseudonym key for this slot, OR
- the ciphertext is an encryption of a *null* cover message, which, when combined with other cover ciphertexts and exactly one actual encrypted message ciphertext, will combine to reveal the encrypted message.

Only the pseudonym key owner can produce a correctness proof for an arbitrary message following the first alternative above, while any node can generate an “honest” cover ciphertext—and the proof by construction reveals no information about *which* alternative the proof generator followed. We leave discussion of further details of this process to Section 5, but merely note that such “either/or” proofs are pervasive and well-understood in the cryptographic theory community.

In Verdict, each client computes and attaches a cryptographic correctness proof to each ciphertext it sends to its upstream server, and each server in turn attaches a correctness proof to the server-side ciphertext it generates in Phase 3 of each round (Figure 3). In principle, therefore, each server can immediately verify the correctness of any client’s or other server’s ciphertext it receives, *before* “accepting” it and combining it with the other ciphertexts for that protocol round. As in Dissent, Verdict achieves resilience to client churn by (optionally) requiring clients to submit their ciphertexts before a certain “deadline” in each messaging round. We describe this technique in Section 4.5.

While Verdict nodes can *in principle* verify the correctness of any received ciphertext immediately, actually doing so has performance costs. These costs lead to design tradeoffs between “eager” and “lazy” verification, both of which we implement and evaluate later in Section 6. Lazy verification enables the critical servers to avoid significant computation costs during rounds that are not disrupted, at the expense of making a round’s output unusable if it *is* disrupted. Even if a lazily-verified round is disrupted, however, the fact that Verdict nodes always generate and transmit signed ciphertext correctness proofs greatly simplifies and shortens the retroactive blame process with respect to Dissent.

Verdict currently treats *server-side* failures of all types, including invalid server ciphertexts, as “major events” requiring administrative action, and simply halts the protocol with an alert until such action is taken. Section 4.6 later discusses approaches to make Verdict resilient against server-side failures, but we leave implementing and evaluating such mechanisms to future work. Such server-side failures affect only availability, however; anonymity remains protected as long as at least one (not necessarily online) server remains uncompromised.

### 4.3 Scheduling Pseudonym Keys

To assign ownership of transmission slots to clients in such a way that *no one* knows which client owns which slot, Verdict applies an architectural idea from Dissent [54]. At the start of a Verdict session, each of the  $N$  clients secretly submits a slot owner pseudonym key to a verifiable shuffle protocol [38] run by the servers. The public output of the shuffle is the  $N$  pseudonym keys in permuted order—such that *no one* knows which node submitted which pseudonym key other than their own. Verdict participants then use each of these  $N$  pseudonym keys to initialize  $N$  concurrent instances of the core Verdict DC-net with each instance becoming a slot in a verifiable DC-net *transmission schedule*.

**Scheduling Policy** Not every client will necessarily want to transmit an anonymous message in every messaging round. In addition, clients may want to transmit

messages of different lengths. To make Verdict more efficient in these cases, Verdict allows clients to request a change in the length of their messaging slot (e.g., so that a client can send a long message in a single messaging round) and to temporarily “close” their transmission slot (if a client does not expect to send a message for several rounds). Clients issue these requests by prepending a few bits of control data to the anonymous message they send in their transmission slot.

### 4.4 Hybrid XOR/Verifiable DC-Nets

While the verifiable DC-net design above may be needed in scenarios in which disruptions are frequent, the public-key cryptography involved imposes a much higher computational cost than traditional XOR-based DC-nets. To offer better performance in groups with fewer or less frequent disruptions, Verdict has a “hybrid” mode of operation that uses the fast XOR-based DC-net when there are no active disruptors in the group, and reverts to a verifiable DC-net in the presence of an active disruptor. This hybrid Verdict DC-net marries the relatively low computational cost of the XOR-based DC-net with the low accountability cost of the verifiable DC-net.

To set up a hybrid DC-net session, all protocol participants first participate in a pseudonym signing key shuffle, as described above in Section 4.3. At the conclusion of the shuffle, all nodes initialize *two* DC-net slots for *each* of the  $N$  clients: one traditional Dissent-style DC-net, and one verifiable Verdict DC-net.

When the group is not being disrupted, clients transmit in their Dissent DC-net slot, allowing nodes to take advantage of the speed of Dissent’s XOR-based DC-net. When nodes detect the corruption of a message in the Dissent DC-net, the client whose message was corrupted reverts to transmitting in its *verifiable* DC-net slot. This client can use the verifiable slot to transmit anonymously the “accusation” necessary to identify the disruptor in the Dissent accusation process [54, Section 3.9]. The Verdict DC-net replaces the expensive verifiable shuffle necessary for nodes to exchange the accusation information in Dissent. In this way, Verdict offers the normal-case efficiency of XOR-based DC-nets while greatly reducing the cost of tracing and excluding disruptors.

### 4.5 Client Churn

In realistic deployments clients may go offline at any time, and this problem becomes severe in large groups of unreliable clients exhibiting constant churn. To prevent slow or unresponsive clients from disrupting communication, the servers need not wait in Phase 2 for all downstream clients to submit ciphertexts. Instead, servers can wait for a fixed threshold of  $t \leq N$  clients to submit ciphertexts, or for some fixed time interval  $\tau$ . Servers might also use some more complicated *window*

*closure policy*, as in Dissent [54]: e.g., wait for a threshold of clients and then an additional time period before proceeding. The participants must agree on a window closure policy before the protocol run begins.

There is an inherent tradeoff between anonymity and the system’s ability to cope with unresponsive clients. If the servers close the ciphertext submission window too aggressively, honest but slow clients might be unable to submit their ciphertexts in time, reducing the anonymity of clients who do manage to submit messages. In contrast, if the servers wait until every client has submitted a ciphertext, a single faulty client could prevent protocol progress indefinitely. Optimal policy choices depend on the security requirements of the application at hand.

## 4.6 Limitations and Future Enhancements

This section outlines some of Verdict’s current limitations, deployment issues, and areas for future work.

**Group Evolution** Verdict’s architecture assumes that, at the start of the protocol, group members agree to a “roster” of protocol participants—essentially a list of public keys defining the group’s membership. The current prototype simplistically assumes that this group roster is a static list, and that the session nonce is a hash of a file containing this roster and other group policy information. This design trivially ensures that all nodes participating in a given group (uniquely identified by its session nonce) agree upon the same roster and policy. Changing the group roster or policy in the current prototype requires forming a new group, but we are exploring approaches to group management which would allow for on-the-fly membership changes. For now, we simply note that Verdict’s security properties critically depend on group membership policy decisions, which affect how quickly adversarial participants can infiltrate a group. We consider group management policy to be orthogonal to this paper’s communication mechanisms.

**Sybil Attacks** If it is too easy to join a group, dishonest participants might flood the group with Sybil identities [20], giving an anonymous slot owner the impression that she has more anonymity than she actually does. The current “static group” design shifts the Sybil attack prevention problem to whomever formulates the group roster. Dynamic group management schemes could leverage existing Sybil prevention techniques [49, 55, 56], but we do not consider such approaches herein.

**Membership Concealment** Verdict considers the group roster, containing the public keys of all participants, to be public information: concealing participation in the protocol is an orthogonal security goal that Verdict currently does not address. We are exploring the use of anonymous authentication techniques [24, 31, 43] to enable Verdict clients to “sign on” and prove member-

ship in the group without revealing to the Verdict servers (or to the adversary) *which* specific group members are online at any given time. Further, we expect that Verdict’s design could be composed with other techniques to achieve membership concealment [35, 51], but we leave such enhancements to future work.

**Unresponsive Servers** Verdict currently assumes that the servers supporting a group are well-provisioned and highly reliable, and the system simply ceases communication progress in the face of any server’s failure. Any fault-masking mechanism would be problematic, in fact, in the face of Verdict’s assumption that only one server might be honest: if that one honest server goes offline and the protocol continues without it, then the remaining dishonest servers could collude against all honest users.

If we assume that there are  $h > 1$  honest servers, however, a currently unimplemented variation of Verdict could allow progress if as many as  $h - 1$  servers are unresponsive. Before the protocol run, every server uses a *publicly verifiable secret sharing scheme* [45], to distribute shares of its per-session secret key. The scheme is configured such that any quorum of  $M - h + 1$  shares is sufficient to reconstruct the secret. Thus, at least one honest server must remain online and contribute a share for a secret to be reconstructed. (Golle and Juels [27] also use a secret-sharing scheme, but they rely on a trusted third-party to generate and distribute the shares.)

If a server becomes unresponsive, the remaining online servers can broadcast their shares of the unresponsive server’s secret key. Once a quorum of servers broadcasts these shares, the remaining online servers will be able to reconstruct the unresponsive server’s private key. Thereafter, each server can simulate the unresponsive server’s messages for the rest of the protocol session.

**Blame Recovery** The current Verdict prototype can detect server misbehavior, but it does not yet have a mechanism by which the remaining servers can collectively form a new group “roster” with the misbehaving nodes removed. We expect off-the-shelf Byzantine Fault Tolerance algorithms [12] to be applicable to this *group evolution* problem. Using BFT to achieve agreement, however, requires a stronger security assumption: in a group with  $f$  dishonest servers, there must be at least  $3f + 1$  total servers. We sketch a possible BFT-based group evolution approach here.

The BFT cluster’s shared state in this case is the group “roster,” containing the session nonce and a list of all active Verdict clients and servers, identified by their public keys. The two operations in this BFT system are:

- `EVOLVE_GROUP(nonce, node_index, proof)`, a request to remove a dishonest node (identified by `node_index`) from the group roster. BFT servers



remove the dishonest node from the group if the proof is valid, yielding the new group roster.

- `GET_GROUP()`, which returns current the group roster. If, at some point during the Verdict session, a Verdict node concludes that the protocol has failed due to the dishonesty of node  $d$ , this honest node makes an `EVOLVE_GROUP` request to the BFT cluster and waits for a response. The honest BFT servers will agree on a new group roster with the dishonest node  $d$  removed and the Verdict servers will begin a new instance of the Verdict protocol with the new group roster. Clients use `GET_GROUP` to learn the new group roster.

## 5 Verifiable DC-net Constructions

The Verdict architecture relies on a verifiable DC-net primitive that has many possible implementations. In this section, we first describe the general interface that each of the cryptographic constructions must implement—which could be described as a “Verdict ciphersuite API”—then we describe the three specific experimental schemes that Verdict currently implements.

All three schemes are founded on standard, well-understood cryptographic techniques that have been formally developed and rigorously analyzed in prior work. As with most practical, complex distributed systems with many components, however, we cannot realistically offer a rigorous proof that these cryptographic tools “fit together” correctly to form a perfectly secure overall system. (This is true even of SSL/TLS and its ciphersuites, which have received far more cryptographic scrutiny than Verdict but in which flaws are still found regularly.) We also make no claim that any of the current schemes are “the right” ones or achieve any particular ideal; we merely offer them as contrasting points in a large design space. To lend some informal credibility to their security, we note that our pairing-based scheme is closely modeled on the verifiable DC-nets scheme that Golle and Juels already developed formally [27], and Appendix C sketches a security argument for the third and most computationally efficient scheme.

### 5.1 Verifiable DC-net Primitive API

The core cryptographic primitive consists of a set of six methods. Each of these six methods takes a list of protocol session parameters (e.g., group roster, session nonce, slot owner’s public key) as an implicit argument:

- *Cover Create*: Given a client session secret key, return a valid client ciphertext representing “cover traffic,” which do not contain actual messages.
- *Owner Create*: Given a client session secret key, the slot owner’s pseudonym secret key, and a plaintext message  $m$  to be transmitted anonymously, return a valid *owner* ciphertext that encodes message  $m$ .

- *Client Verify*: Given a client public key and a client ciphertext, return a boolean flag indicating whether the client ciphertext is valid.
- *Server Create*: Given a server private key and a set of client ciphertexts, return a valid server ciphertext.
- *Server Verify*: Given a server public key, a set of valid client ciphertexts, and a server ciphertext, return a flag indicating whether the server ciphertext is valid.
- *Reveal*: Combine  $N$  client ciphertexts and  $M$  server ciphertexts, returning the plaintext message  $m$ .

However these methods are implemented, they must obey the following security properties, stated informally:

- **Completeness**: An honest verifier always accepts a ciphertext generated by an honest client or server.
- **Soundness**: With overwhelming probability an honest verifier rejects an incorrect ciphertext, such as an owner ciphertext formed without knowledge of the owner’s pseudonym secret key.
- **Zero-knowledge**: A verifier learns nothing about a ciphertext besides the fact that it is correctly formed.
- **Integrity**: Combining  $N$  valid client ciphertexts, including one ciphertext from the anonymous slot owner, and  $M$  valid server ciphertexts, reveals the slot owner’s plaintext message.
- **Anonymity**: A verifier cannot distinguish a client ciphertext from the anonymous slot owner’s ciphertext. Appendix C offers a game-based definition of anonymity.

In practice, each of our current implementations of this verifiable DC-nets primitive achieve these security properties in the random-oracle model [5] using non-interactive zero-knowledge proofs [28].

### 5.2 ElGamal-Style Construction

The first scheme builds on the ElGamal public-key cryptosystem [21]. In ElGamal, a public/private keypair has the form  $\langle B, b \rangle = \langle g^b, b \rangle$ ,<sup>1</sup> and plaintexts and ciphertexts are elements of an algebraic group  $G$ .<sup>2</sup> We refer to this as the “ElGamal-style” construction because its use of an ephemeral public key and encryption by multiplication structurally resembles the ElGamal cryptosystem. Our construction does *not* exhibit the malleability of textbook ElGamal encryption, however, because a proof of knowledge of the secret ephemeral public key is attached to every ciphertext element.

**Client Ciphertext Construction** Given a list of server public keys  $\langle B_1, \dots, B_M \rangle$ , a client constructs a ciphertext

<sup>1</sup> We do not require that a trusted third party generate participants’ keypairs, but we *do* require participants to prove knowledge of their secret key at the start of a protocol session, for reasons described in Appendix A.

<sup>2</sup> Throughout, unless otherwise noted, group elements are members of a finite cyclic group  $G$  in which the Decision Diffie-Hellman (DDH) problem [6] is assumed computationally infeasible, and  $g$  is a public generator of  $G$ .

by selecting an ephemeral public key  $R_i = g^{r_i}$  at random and computing the ciphertext element:

$$C_i = m \left( \prod_{j=1}^M B_j \right)^{r_i}$$

If the client is the slot owner, the client sets  $m$  to its secret message, otherwise the client sets  $m = 1$ .

To satisfy the security properties described in Section 5.1, the client must somehow *prove* that the ciphertext tuple  $\langle R_i, C_i \rangle$  was generated correctly. We adopt the technique of Golle and Juels [27] and use a non-interactive proof-of-knowledge of discrete logarithms [11] to prove that the ciphertext has the correct form. If the slot owner’s pseudonym public key is  $Y$ , the client’s ephemeral public key is  $R_i$ , and the client’s ciphertext element is  $C_i$ , the client generates a proof:

$$\text{PoK}\{r_i, y : (R_i = g^{r_i} \wedge C_i = (\prod_{j=1}^M B_j)^{r_i}) \vee Y = g^y\}$$

In words: the sender demonstrates that *either* it knows the discrete logarithm  $r_i$  of the ephemeral public key  $R_i$ , and the ciphertext is the product of all server public keys raised to the exponent  $r_i$ ; *or* the sender knows the slot owner’s secret pseudonym key  $y$ , in which case the slot owner can set  $C_i$  to a value of her choosing. Appendix D details how to construct and verify this type of non-interactive zero-knowledge proof.

Note that a dishonest slot owner can set  $C_i$  to a maliciously constructed value (e.g.,  $C_i = 1$ ). The only effect of such an “attack” is that the slot owner compromises *her own* anonymity. Since a dishonest slot owner can always compromise her own anonymity (e.g., by publishing her secret keys), a dishonest slot owner achieves nothing by setting  $C_i$  maliciously.

The tuple  $\langle R_i, C_i, \text{PoK} \rangle$  serves as the client’s ciphertext. As explained in Section 4.1, all participants sign the messages they exchange for accountability.

**Server Ciphertext Construction** Given a server public key  $B_j = g^{b_j}$  and a list of ephemeral client public keys  $\langle R_1, \dots, R_N \rangle$ , server  $j$  generates its server ciphertext as:

$$S_j = \left( \prod_{i=1}^N R_i \right)^{-b_j}$$

The server proves the validity of its ciphertext by creating a non-interactive proof of knowledge that it knows its secret private key  $b_j$  and that its ciphertext element  $S_j$  is the product of the ephemeral client keys raised to the exponent  $-b_j$ :

$$\text{PoK}\{b_j : B_j = g^{b_j} \wedge S_j = (\prod_{i=1}^N R_i)^{-b_j}\}$$

**Message Reveal** To reveal the plaintext message, a participant computes the product of  $N$  client ciphertext elements and  $M$  server ciphertext elements:

$$m = \left( \prod_{i=1}^N C_i \right) \left( \prod_{j=1}^M S_j \right)$$

Each factor  $g^{r_i b_j}$ , where  $r_i$  is client  $i$ ’s ephemeral secret key and  $b_j$  is server  $j$ ’s secret key, is included exactly

twice in the above product—once with a positive sign in the client ciphertexts and once with a negative sign in the server ciphertexts. These values therefore cancel, leaving only the plaintext  $m$ .

**Drawbacks** Since the clients must use a new ephemeral public key for *each* ciphertext element, sending a plaintext message that is  $L$  group elements in length requires each client to generate and transmit  $L$  ephemeral public keys. The proof of knowledge for this construction is  $L + O(1)$  group elements long, so a message of  $L$  group elements expands to  $3L + O(1)$  elements.

### 5.3 Pairing-Based Construction

A major drawback of the ElGamal construction is that, due to the need for ephemeral keys, every ciphertext is three times as long as the plaintext it encodes. Golle and Juels [27] use bilinear maps to eliminate the need for ephemeral keys. Our pairing-based construction adopts elements of their technique, while avoiding their reliance on a trusted third party, a secret-sharing scheme, and a probabilistic transmission scheduling algorithm.

A symmetric bilinear map  $\hat{e}$  maps two elements of a group  $G_1$  into a target group  $G_2$ — $\hat{e} : G_1 \times G_1 \rightarrow G_2$ . A bilinear map has the property that:  $\hat{e}(aP, bQ) = \hat{e}(P, Q)^{ab}$ .<sup>3</sup> To be useful, the map must also be non-degenerate (if  $P$  is a generator of  $G_1$ ,  $\hat{e}(P, P)$  is a generator of  $G_2$ ) and efficiently computable [8]. We assume that the decision bilinear Diffie-Hellman assumption [7] holds in  $G_1$ .<sup>4</sup>

Since pairing allows a *single* pair of public keys to generate a *sequence* of shared secrets, clients need not generate ephemeral public keys for each ciphertext element they send. This optimization leads to shorter ciphertexts and shorter correctness proofs.

**Client Ciphertext Construction** For a set of server public keys  $\langle B_1, \dots, B_M \rangle$ , a public nonce  $\tau \in G_1$  computed using a hash function, and a client public key  $A_i = g^{a_i}$ , a pairing-based client ciphertext has the form:

$$C_i = m \hat{e} \left( \prod_{j=1}^M B_j, \tau \right)^{a_i}$$

As before, if the client is not the slot owner, the client sets  $m = 1$ . Each client can produce a proof of the correctness of its ciphertext by executing a proof of knowledge similar to one used in the ElGamal-style construction above:

$$\text{PoK}\{a_i, y : (A_i = g^{a_i} \wedge C_i = \hat{e}(\prod_{j=1}^M B_j, \tau)^{a_i}) \vee Y = g^y\}$$

While the ElGamal-style scheme requires  $3L + O(1)$  group elements to encode  $L$  elements of plaintext, a

<sup>3</sup> Since  $G_1$  is usually an elliptic curve group, the generator of  $G_1$  is written as  $P$  (an elliptic curve point) and the repeated group operation is written as  $aP$  instead of  $g^a$ . We will use the latter notation for consistency with the rest of this section.

<sup>4</sup> Note that the decision Diffie-Hellman problem is easy in  $G_1$ , since given  $g, g^a, g^b, g^c \in G_1$ , a DDH tuple will always satisfy  $\hat{e}(g^a, g^b) = \hat{e}(g, g^c)$  if  $c = ab \pmod q$ .

pairing-based ciphertext requires only  $L + O(1)$  group elements to encode an  $L$ -element plaintext.

**Server Ciphertext Construction** Using a server public key  $B_j = g^{b_j}$ , a public round nonce  $\tau$ , and client public keys  $\langle A_1, \dots, A_N \rangle$ , a server ciphertext has the form:

$$S_j = \hat{e}(\prod_{i=1}^N A_i, \tau)^{-b_j}$$

The server proof of correctness is then:

$$\text{PoK}\{b_j : B_j = g^{b_j} \wedge S_j = \hat{e}(\prod_{i=1}^N A_i, \tau)^{-b_j}\}$$

**Message Reveal** To reveal the plaintext, the servers take the product of all client and server ciphertexts:

$$m = (\prod_{i=1}^N C_i)(\prod_{j=1}^M S_j)$$

**Drawbacks** The main downside of this construction is the relatively high computational cost of the pairing operation. Computing the pairing operation on two elements of  $G_1$  can take an order of magnitude longer than a normal elliptic curve point addition in a group of similar security level, as Section 6.2 below will show.

## 5.4 Hashing-Generator Construction

Our hashing-generator construction pursues a “best of both worlds” combination of the ElGamal-style and pairing-based constructions. This construction has short ciphertexts, like the pairing-based construction, but avoids the computational cost of the pairing-based scheme by using conventional integer or elliptic curve groups. To achieve both of these desired properties, the hashing-generator construction adds some protocol complexity, in the form of a session set-up phase.

**Set-up Phase** In the set-up phase, each client  $i$  establishes a Diffie-Hellman shared secret  $r_{ij}$  with every server  $j$  using their respective public keys  $g^{a_i}$  and  $g^{b_j}$  by computing  $r_{ij} = \text{KDF}(g^{a_i b_j})$  using a key derivation function KDF. Clients publish commitments to these shared secrets  $R_{ij} = \hat{g}^{r_{ij}}$  using another public generator  $\hat{g}$ .

The hashing-generator construction requires a process by which participants compute a sequence of generators  $g_1, \dots, g_L$  of the group  $G$ , such that no participant knows the discrete logarithm of any of these generators with respect to any other generator. In other words, *no one* knows an  $x$  such that  $g_i^x = g_j$ , for any  $i, j$  pair. In practice, participants compute this sequence of generators by hashing a series of strings, (e.g., the round nonce concatenated with “1”, “2”, “3”, ...), to choose the set of generating group elements.

At the end of the set-up phase, every client  $i$  can produce a *sequence* of shared secrets with each server  $j$  using their shared secret  $r_{ij}$  and the  $L$  generators:  $g_1^{r_{ij}}, \dots, g_L^{r_{ij}}$ . In the  $\ell$ th message exchange round, all participants use generator  $g_\ell$  as their common generator.

**Client Ciphertext Construction** To use the hashing-generator scheme to create a ciphertext, the client uses its shared secrets  $r_{i1}, \dots, r_{iM}$  with the servers, and generator  $g_\ell$  for the given protocol round to produce a ciphertext:

$$C_i = m g_\ell^{(\sum_{j=1}^M r_{ij})}$$

As before,  $m = 1$  if the sender is not the slot owner.

To prove the validity of a ciphertext element, the client executes the following proof of knowledge, where  $Y$  is the slot owner’s pseudonym public key,  $r_i = \sum_{j=1}^M r_{ij}$ , and  $R_{ij}$  is the commitment to the secret shared between client  $i$  and server  $j$ :

$$\text{PoK}\{r_i, y : ((\prod_{j=1}^M R_{ij}) = \hat{g}^{r_i} \wedge C_i = g_\ell^{r_i}) \vee Y = g^y\}$$

**Server Ciphertext Construction** Server  $j$ ’s ciphertext for the  $\ell$ th message exchange round is similar to the client ciphertext, except with negated exponents:

$$S_j = g_\ell^{(-\sum_{i=1}^N r_{ij})}$$

The server proves correctness of a ciphertext by executing a proof of knowledge, where  $r_j = \sum_{i=1}^N r_{ij}$ :

$$\text{PoK}\{r_j : (\prod_{i=1}^N R_{ij}) = \hat{g}^{r_j} \wedge S_j = g_\ell^{-r_j}\}$$

**Message Reveal** The product of the client and server ciphertexts reveals the slot owner’s plaintext message  $m$ :

$$m = (\prod_{i=1}^N C_i)(\prod_{j=1}^M S_j)$$

**Failed Session Set-up** A dishonest client  $i$  might try to disrupt the protocol by publishing a corrupted commitment  $R'_{ij}$  that disagrees with server  $j$ ’s commitment  $R_{ij}$  to the shared secret  $r_{ij} = \text{KDF}(g^{a_i b_j})$ . If the commitments disagree, the honest server can prove its innocence by broadcasting the Diffie-Hellman secret  $\rho_{ij} = g^{a_i b_j}$  along with a proof that it correctly computed the Diffie-Hellman secret using its public key  $B_j$  and the client’s public key  $A_i$ .

$$\text{PoK}\{b_j : \rho_{ij} = A_i^{b_j} \wedge B_j = g^{b_j}\}$$

If the server is dishonest, the client can produce a similar proof of innocence. Any user can verify this proof, and then use  $g^{a_i b_j}$  to recreate the correct commitment  $R_{ij}$ . Once the verifier has the correct commitment  $R_{ij}$ , the verifier can confirm either that the client in question published an invalid commitment or that the server in question dishonestly accused the client.

Since the session set-up between client  $i$  and server  $j$  will only fail if either  $i$  or  $j$  is dishonest, there is no security risk to publishing the shared secret  $g^{a_i b_j}$  after a failed set-up—the dishonest client (or server) could have shared this secret with the adversary anyway.

**Long Messages** The client and server ciphertext constructions described above allow the slot owner to transmit a plaintext message  $m$  that is at most one group element in length in each run of the protocol. To encode

longer plaintexts efficiently, participants use a modified proof-of-knowledge construction that proves the validity of  $L$  ciphertext elements ( $C_{i,1}$  through  $C_{i,L}$ ) at once:

$$\text{PoK}\{r_i, y : ((\prod_{j=1}^M R_{ij} = \hat{g}^{r_i}) \wedge (\wedge_{\ell=1}^L C_{i,\ell} = g_\ell^{r_i})) \vee Y = g^y\}$$

Servers can use a similarly modified proof of knowledge. This modified knowledge proof is surprisingly compact: the length of the proof is *constant* in  $L$ , since the length of the proof is linear in the number of proof variables (here, the only variables are  $r_i$  and  $y$ ). The total length of the tuple  $\langle \tilde{C}_i, \text{PoK} \rangle$  using this proof is  $L + O(1)$ .

**Lazy Proof Verification** In the basic protocol, every server verifies the validity proof on every client ciphertext in every protocol round. To avoid these expensive verification operations, servers can use *lazy proof verification*: servers check the validity of the client proofs only if they detect, at the end of a protocol run, that the anonymous slot owner’s message was corrupted. For reasons discussed in Appendix E, lazy proof verification is possible only using the pairing-based or hashing-generator ciphertext constructions.

**Security Analysis** Since the hashing-generator scheme is the most performant variant, we sketch an informal security proof for the hashing-generator proof construction in Appendix C.

## 6 Evaluation

This section describes our Verdict prototype implementation and summarizes the results of our evaluations.

### 6.1 Implementation

We implemented the Verdict protocol in C++ using the Qt framework as an extension to the existing Dissent prototype [54]. Our implementation uses OpenSSL 1.0.1 for standard elliptic curve groups, Crypto++ 5.6.1 for big integer groups, and the Stanford Pairing-Based Cryptography (PBC) 0.5.12 library for pairings [50]. Unless otherwise noted, the evaluations use 1024-bit integer groups, the 256-bit NIST P-256 elliptic curve group [39], and a pairing group in which  $G_1$  is an elliptic curve over a 512-bit field (using PBC’s “Type A” parameters) [32]. We collected the macrobenchmark and end-to-end evaluation results on the DeterLab [18] testbed.

The source code for our implementation is available at <https://github.com/DeDis/Dissent>.

### 6.2 Microbenchmarks

To compare the pure computational costs of the different DC-net schemes, Figure 5 shows ciphertext generation and verification throughput measured at a variety of block sizes, running on a workstation with a 3.2 GHz Intel Xeon W3565 processor. These experiments involve no network activity, and are single-threaded, thus they do not reflect any speedup that parallelization might offer.

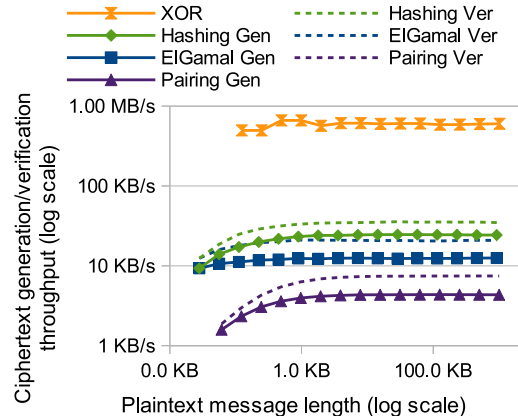


Figure 5: Ciphertext generation and verification throughput for the three verifiable DC-net variants and the XOR-based scheme.

The hashing-generator construction, which is the fastest scheme tested, encrypts 20 KB of client plaintext per second. The slowest, pairing-based construction encrypts around 3 KB per second. The fastest verifiable scheme is still over an order of magnitude slower than the traditional (unverifiable) XOR-based scheme, which encrypts 600 KB of plaintext per second. The hashing-generator scheme performs best because it needs no pairing operations and requires fewer group exponentiations than the ElGamal construction.

Figure 5 shows that ciphertext verification is slightly faster than ciphertext generation. This is because generating the ciphertext and zero-knowledge proof requires more group exponentiations than proof verification does.

The three constructions also vary in the size of ciphertexts they generate (Figure 6). While the pairing-based scheme and the hashing-generator schemes encrypt length  $L$  plaintexts as ciphertexts of length  $L + O(1)$ , the ElGamal-style scheme encrypts length  $L$  plaintexts as length  $3L + O(1)$  ciphertexts. As discussed in Section 5.2, for every plaintext message element encrypted, ElGamal-style ciphertexts must include an ephemeral public key and an additional proof-of-knowledge group element. Since the hashing-generator scheme is the fastest and avoids the ElGamal scheme’s ciphertext expansion, subsequent experiments use the hashing-generator scheme unless otherwise noted.

### 6.3 Accountability Cost

Figure 7 presents three graphs: (a) the time it takes to set up a transmission schedule via a verifiable shuffle, prior to DC-net communication, (b) the time required to execute a single DC-net protocol round in each scheme, and (c) the time required to identify a disruptor. The graphs compare four protocol variants: Dissent, Verdict, Verdict

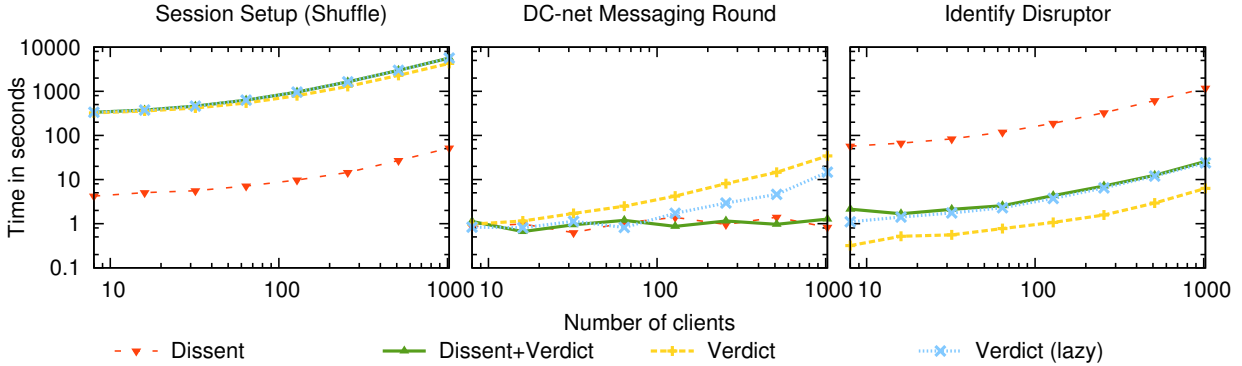


Figure 7: Time required to initialize a session, perform one messaging round, and to identify a disruptor.

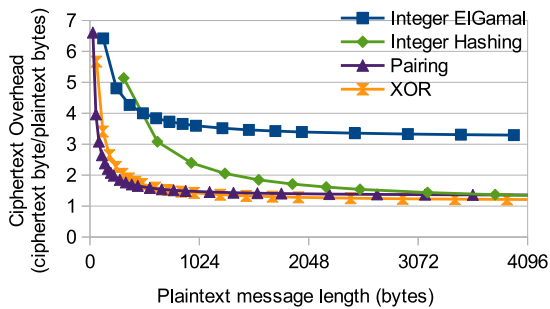


Figure 6: Ciphertext expansion factor (overhead) using the integer ElGamal-style, pairing-based, and hashing-generator protocol variants.

with lazy proof verification, and the Dissent+Verdict hybrid DC-net. We ran this experiment on DeterLab using 8 servers and 128 clients. To scale beyond 128 clients, we ran multiple client processes on each client machine. Session setup time measures the time from session start to just before the first DC-net messaging round.

The one-time session setup time for Verdict is longer than for Dissent because the verifiable shuffle implementation Dissent uses is heavily optimized for shuffling DSA signing keys. Shuffling Verdict public keys, which are drawn from different group types, requires using a less-optimized version of the verifiable shuffle. We do not believe this cost is fundamental to the Verdict approach, and in any case these setup costs are typically amortized over many DC-net rounds.

The Dissent+Verdict hybrid DC-net is just as fast as Dissent in the normal case, since Dissent and the hybrid DC-net run *exactly the same code* if there is no active disruptor in the group. Network latency comprises the majority of the time for a messaging round when using the Dissent and the hybrid Dissent+Verdict DC-nets—messaging rounds take between 0.6 and 1.4 seconds to complete in network sizes of 8 to 1,024 clients.

In contrast, Verdict becomes computationally limited at 64 clients, taking approximately 2.5 seconds per round. Verdict (lazy) improves upon this by becoming computationally limited at 256 clients, requiring approximately 3 seconds per messaging round.

Verdict incurs the lowest accountability (blame) cost of the four schemes. Verdict’s verifiable DC-net checks the validity of each client ciphertext before processing it further, so the time-to-blame in Verdict is equal to the cost of verifying the validity proofs on  $N$  client ciphertexts. “Verdict (lazy)” uses the lazy proof verification technique described in Section 5.4—servers verify the client proofs of correctness only if they detect a disruption. Lazy proof verification delays the verification operation to the end of a messaging phase, so the time-to-blame is slightly higher than in pure Verdict.

Dissent, which has the highest time-to-blame, has an accountability process that requires the anonymous client whose message was corrupted to submit an “accusation” message to a lengthy verifiable shuffle protocol, in which all members participate. This verifiable shuffle is the reason that Dissent takes the longest to identify a disruptor. The hybrid Dissent+Verdict DC-net (Section 4.4) avoids Dissent’s extra verifiable shuffle by falling back instead to a verifiable DC-net to resolve disruptions.

As Figure 7 shows, the messaging round time in the hybrid Dissent+Verdict DC-net is as fast as in Dissent, but the hybrid scheme reduces Dissent’s time to detect misbehavior by roughly two orders of magnitude.

## 6.4 Anonymous Microblogging

Verdict’s ability to tolerate many dishonest nodes makes it potentially attractive for anonymous microblogging in groups of hundreds of nodes. In Twitter, messages have a maximum length of 140 bytes, which means that a single tweet can fit into a few 256-bit elliptic curve group elements. Twitter users can also tolerate messaging latency

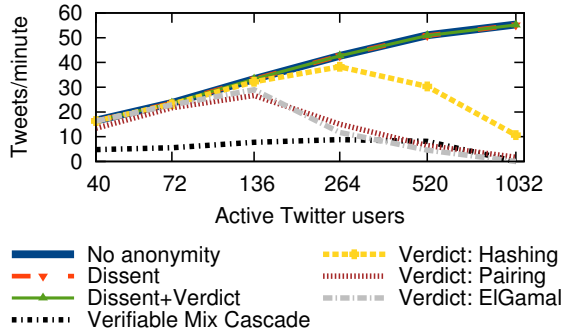


Figure 8: Rate at which various anonymity schemes process tweets, for varying numbers of active users.

of tens of seconds or even a few minutes, which would be unacceptable for interactive web browsing.

This experiment evaluates the suitability of Verdict for *small-scale* anonymous microblogging applications, giving users anonymity among hundreds of nodes, e.g., for students microblogging on a university campus. To test Verdict in this scenario, we recorded 5,000 Twitter users’ activity for one-hour and then took subsets of this trace: the smallest subset contained only the Tweets of the 40 most active users, and the largest subset contained the Tweets of the 1,032 most active users. We replayed each of these traces through Dissent and through Verdict, using each of the three ciphertext constructions.

We ran our experiment on DeterLab [18], on a test topology consisting of eight servers connected to a 100 Mbps LAN with 10 ms of server-to-server latency, and with each set of clients connecting to their upstream server over a *shared* 100 Mbps link with 50 ms of latency. Scarcity of testbed resources limited the number of available delay links, but our experiment attempts to approximate a wide-area deployment model in which clients are geographically dispersed and bandwidth-limited.

Figure 8 shows the Tweet-rate latency induced by the different anonymity systems relative to the baseline (no anonymity) as the number of active users (and hence, the anonymity set size) in the trace increases. Both Dissent and the Dissent+Verdict hybrid systems can keep pace with the baseline in a 1,000-node network—the largest network size feasible on our testbed. The pure Verdict variants could not keep pace with the baseline in a 1,000-node network, while hashing-generator variant of Verdict runs almost as quickly as the baseline in an anonymity set size of 264. These results suggest that Verdict might realistically support proactively accountable anonymity for microblogging groups of up to hundreds of nodes.

Figure 8 also compares Verdict to a mix-net cascade (a set of mix servers) in which each mix server uses a Neff proof-of-knowledge [38] to demonstrate that it has

performed the mixing operation properly. Like Verdict, this sort of mix cascade forms a traffic-analysis-resistant anonymity system, so it might be used as an alternative to Verdict for anonymous messaging. Our evaluation results demonstrate that the hashing-generator variant of Verdict outperforms the mix cascade at all network sizes and that the Tweet throughput of the Dissent+Verdict hybrid is more than  $6\times$  greater than the throughput of the mix cascade at a network size of 564 participants.

## 6.5 Anonymous Web Browsing

Dissent demonstrated that accountable DC-nets are fast enough to support anonymous interactive Web browsing in local-area network deployments [54]. We now evaluate whether Verdict is similarly usable in a web browsing scenario. Our experiment simulates a group of nodes connected to a single WLAN network. This configuration emulates, for example, a group of users in an Internet café browsing the Internet anonymously.

In our simulation on DeterLab [18], 8 servers and 24 clients communicate over a network of 24 Mbps links with 20 ms node-to-node latency. To simulate a Web browsing session, we recorded the sequence of requests and responses that a browser makes to download home page content (HTML, CSS files, images, etc.) from the Alexa “Top 100” Web pages [2]. We then replayed this trace with the client using one of four anonymity overlays: no anonymity, the Dissent DC-net, the Verdict-only DC-net, and the Dissent+Verdict hybrid DC-net. The simulated client sends the upstream (request) traffic through the anonymity network and servers broadcast the downstream (response) traffic to all nodes.

Figure 9 charts the time required to download all home page content using the four different network configurations. The median Web page took one second to load with no anonymity, fewer than 10 seconds over Dissent, and around 30 seconds using Verdict only (Figure 10). Notably, the hybrid Dissent+Verdict scheme exhibits performance nearly identical to that of Dissent alone, since it falls back to the slower verifiable Verdict DC-net only when there is active disruption. The Verdict-only DC-net is much slower than Dissent because every node must generate a computationally expensive zero-knowledge proof in every messaging round.

These experiments show that Verdict adds no overhead to Dissent’s XOR-based DC-net in the absence of disruption. In addition, these experiments illustrate the flexibility of verifiable DC-nets, which can be used either as a “workhorse” for anonymous communication or more selectively in combination with traditional XOR-based DC-nets; we suspect that other interesting applications will be discovered in the future.

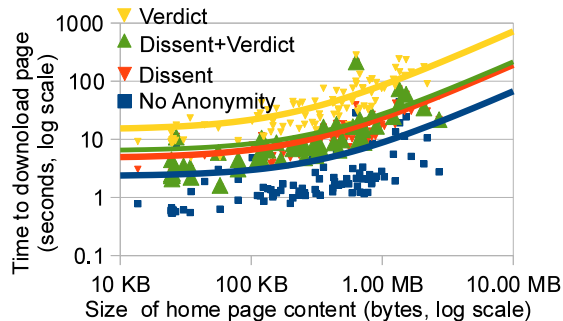


Figure 9: Time required to download home page context for Alexa “Top 100” Web sites (with linear trend lines).

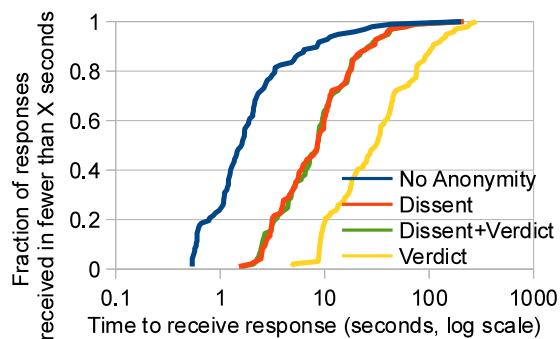


Figure 10: CDF of time required to download home page context for Alexa “Top 100” Web sites.

## 7 Related Work

Chaum recognized the risk of anonymous disruption attacks in his original formulation of DC-nets [13], and proposed a probabilistic tracing approach based on *traps*, upon which Waidner and Pfitzmann expanded [52].

Herbivore [26, 46] sidestepped the disruption issue by forming groups dynamically, enabling nodes to leave disrupted groups and form new groups until they find a disruption-free group. Unfortunately, the likelihood that a group contains some malicious node likely increases rapidly with group size, and hence anonymity set, limiting this and related partitioning approaches [1] to systems supporting small anonymity sets. Further, in an analog to a known attack against Tor [9], an adversary might selectively disrupt only groups he has only *partially* but not *completely* compromised. With a powerful adversary controlling many nodes, after some threshold a victim becomes *more* likely to “settle into” a group that works precisely because it is *completely* compromised, than to find a working uncompromised group.

$k$ -anonymous message transmission [1] also achieves disruption resistance by partitioning participants into small disruption-free groups. A crucial limitation of the  $k$ -anonymity system is that an honest client is anony-

mous *only* among a small constant ( $k$ ) number of nodes. In contrast, Verdict clients in principle obtain anonymity among the set of *all* honest clients using the system.

Dissent [15, 54] uses verifiable shuffles [10, 38] to establish a *transmission schedule* for DC-nets, enabling groups to guarantee a one-to-one correspondence of group members to anonymous transmission slots. The original Dissent protocol [15] offered accountability but limited performance. A more recent version [54] improves performance and scalability, but uses a retrospective “blame” protocol which requires an expensive shuffle when disruption is detected.

Golle and Juels [27] introduced the verifiable DC-net concept and formally developed a scheme based on bilinear maps, forming Verdict’s starting point. To our knowledge this scheme was never implemented in a working anonymous communication system, however, and we find that its expensive pairing operations limit its practical performance.

Crowds [42], LAP [29], Mixminion [17], Tarzan [23], and Tor [19], provide anonymity in large networks, but these systems cannot protect against adversaries that observe traffic [4, 37] or perform active attacks [9] on a large fraction of network links. Verdict maintains its security properties in the presence of this type of strong adversary. A cascade of cryptographically verifiable shuffles [25, 38] can offer the same security guarantees that Verdict does, but these shuffles generally require more expensive proofs-of-knowledge.

## 8 Conclusion

Verdict is a new anonymous group messaging system that combines the traffic analysis resistance of DC-nets with disruption resistance based on public-key cryptography and knowledge proofs. Our experiments show that Verdict may be suitable for messaging in groups of hundreds to thousands of users, and can be combined with traditional XOR-based DC-nets to offer good normal-case performance while reducing the system’s vulnerability to disruption events by two orders of magnitude.

## Acknowledgments

We wish to thank Aaron Johnson, Ewa Syta, Michael J. Fischer, Michael Z. Lee, Michael “Fitz” Nowlan, and Ramki Gummadi for their helpful comments. We also thank our shepherd, Micah Sherr, and the anonymous USENIX reviewers, for their valuable feedback. Finally, we thank the DeterLab staff for their flexibility, patience, and support during the evaluation process. This material is based upon work supported by the Defense Advanced Research Agency (DARPA) and SPAWAR Systems Center Pacific, Contract No. N66001-11-C-4018.

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## A Maliciously Crafted Public Keys

The protocol construction does *not* assume the existence of a trusted third party who generates participants public/private keypairs. Instead, every client must verify the validity of every server’s public key  $g^a$  at the start of a protocol run, and every server must verify the validity of every client’s public key as well.

Participants can verify that a public key  $g^a$  is valid by confirming that  $g^a \in G$ . However, this check is not enough to prevent malicious participants from disrupting a protocol run. For example, if an honest server published a public key  $A = g^a$ , a dishonest server could publish a public key  $B = (g^b)(g^a)^{-1} = g^{b-a}$  whose corresponding secret key is a function of  $A$ ’s secret key. If a client then uses the ElGamal-style ciphertext construction to create a ciphertext using ephemeral private key  $r$  encrypted for both servers, the product of the two servers’ public keys will result in an encryption that server  $B$  can decrypt unilaterally:

$$C = m(AB)^r = m((g^a)(g^b)(g^a)^{-1})^r = mg^{rb}$$

By creating a public key in this way, server  $B$  can “cancel out” the effect of  $A$ ’s public key even without  $A$ ’s cooperation.

To prevent this sort of attack, we require participants to prove knowledge of the discrete logarithm of their public key. In the notation of Camenisch and Stadler, participant must prove:

$$\text{PoK}\{a : A = g^a\}$$

Requiring this proof defeats the attack outline above, because without knowledge of  $A$ ’s secret key  $a$  server  $B$  cannot prove knowledge of the secret exponent  $b - a$ . Since we use public keys of the form  $g^a$  in each of the three ciphertext constructions, it is possible to execute this proof of knowledge regardless of which scheme is in use.

## B Security of Messaging Protocol

This section sketches a security argument that the messaging protocol described in Section 4 satisfies the security properties of integrity, anonymity, and accountability, provided that the underlying cryptographic primitives (described in Section 5) are correct.

### B.1 Integrity

Since every protocol message is signed with the sender’s long-term signing key, and since each message includes a

round nonce unique to this particular run of the protocol, replay and impersonation attacks are infeasible.

Assume, by way of contradiction, that some honest client  $h$  concludes that the protocol run has terminated in success *and* that  $h$  holds a final output message  $m'$  that is unequal to the slot owner’s plaintext message  $m$ .

For  $h$  to conclude that the protocol run succeeded,  $h$  would have had to receive a valid signature on the invalid ciphertext  $m'$  from each of the  $M$  servers in Phase 6

Since at least one of the  $M$  servers is honest, at least one of the servers (call this server  $s$ ), verified each of the proofs of correctness on each of the server ciphertexts in Phase 4. Since (by assumption) the ciphertext construction is sound, each of the server ciphertexts must therefore be a correctly formed server ciphertext (except with negligible probability) corresponding to the client ciphertext set that  $s$  received in Phase 5.

In addition, honest server  $s$  has verified the proof of correctness on each of the client ciphertexts (Phase 3), and the ciphertext soundness property (Section 5.1) means that each of these ciphertexts is also validly constructed.

Since the plaintext element  $m'$  that  $h$  receives is the product of  $N$  valid client ciphertexts, and of  $M$  valid server ciphertexts—one from each server and constructed in response to the client ciphertext set—the product of these ciphertexts will be the slot owner’s original plaintext  $m$ . Since we have  $m = m'$ , but  $m \neq m'$  by assumption, this is a contradiction.

### B.2 Anonymity

The anonymity of the overall protocol derives directly from the anonymity of the underlying cryptographic construction (Section 5). To break the anonymity of the system, an adversary must gain some advantage in distinguishing the slot owner from other clients participating in the protocol. Since the only difference in behavior between the slot owner and the remaining clients comes in Phase 1 of the protocol (in which participants generate their ciphertext messages), the adversary must be able to use the ciphertext messages alone to distinguish the slot owner from others. However, since we assume that the underlying cryptographic primitive maintains slot owner anonymity, the attacker has no feasible way to distinguish the slot owner’s ciphertext from the remaining ciphertexts. We discuss the specific indistinguishability assumptions used in each cryptographic construction in Section 5.

### B.3 Accountability

The accountability property requires that if an honest node concludes that a protocol run has terminated in failure, it holds a third-party verifiable proof of (at least) one dishonest node’s misbehavior.

We enumerate the ways in which a node can misbehave and demonstrate how an honest node can detect each type of misbehavior:

- **Invalid public key (session set-up)** A client or server who submits an invalid public key or an invalid proof of knowledge during the session set-up process will be immediately exposed by the recipient, and the message containing the invalid key becomes proof of the key generator’s dishonesty.

The proof of knowledge ensures that a dishonest node will be unable to pass off an honest node’s public key as his own (since the dishonest node will be unable to produce a signature of his own long-term public key with the honest node’s public key).

- **Invalid client ciphertext (Phase 1)** A client who submits an invalid ciphertext in Phase 1 will be exposed by the receiving server in Phase 2 (if the receiving server is honest) when the server checks the validity of the client’s ciphertext. The signed, invalid ciphertext submitted by the client is the proof of the client’s dishonesty. If the *server* is dishonest, the server will be exposed later in the protocol.
- **Client equivocation (Phase 1)** A client who submits two different ciphertexts to two different servers will be exposed by an honest server in Phase 3 when the server observes the two different valid signed client ciphertexts in two servers’ client ciphertext sets. These two ciphertexts will become the proof of the client’s dishonesty.
- **Server accepts invalid ciphertext (Phase 2)** If a server accepts an *invalid* ciphertext from a client, an honest server will detect this dishonesty in Phase 3, since the dishonest server’s ciphertext set will not match the honest server’s ciphertext set (since honest servers will only transmit a ciphertext set if all client ciphertexts are valid). The honest server will use the dishonest server’s ciphertext set as its proof of the server’s dishonesty.
- **Invalid server ciphertext (Phase 3)** An honest server in Phase 4 will expose a dishonest server that has broadcasted an invalid server ciphertext in Phase 3. The invalid server ciphertext serves as the proof of the server’s dishonesty.
- **Invalid server signature (Phase 4)** Since, in Phase 4, all servers hold the same set of valid client ciphertexts, and a set of  $M$  valid server ciphertexts, each honest server will be able to recover the same plaintext message. An honest server in Phase 5 will expose a dishonest server that has broadcasted an invalid signature in Phase 4 when the honest server checks the dishonest

server’s signature against the revealed plaintext. The invalid signature serves as proof of the server’s dishonesty.

- **Corrupted signature from server (Phase 5)** A server that transmits invalid signatures to its downstream clients in Phase 5 will be revealed by honest clients in Phase 6. Since an honest server will verify each signature before sending them to its downstream clients, an honest client can conclude that if any of the signatures on the plaintext is invalid, its upstream server is dishonest.
- **Un-parseable message (all phases)** If an honest participant ever receives an un-parseable message from another participant, that carries a valid signature with the sender’s long-term signing key, the signed message becomes proof of the dishonest node’s misbehavior.

## C Security Arguments

This section provides a game-based definition of the anonymity property (introduced in Section 5.1) and then argues for the security of the hashing-generator ciphertext construction (introduced in Section 5.4).

### C.1 Anonymity Game

We say that a protocol maintains *slot owner anonymity* if the advantage of any polynomial-time adversary in the following anonymity game is negligible (in the implicit security parameter). The game, which takes place between an adversary and a challenger, proceeds as follows:

- The challenger picks per-session keypairs for the slot owner, for each of the  $M$  servers, and for each of the  $N$  clients.
- The challenger sends to the adversary:
  - all of the public keys,
  - the private keys for the  $M - 1$  dishonest servers, and
  - the private keys for the  $N - 2$  dishonest clients.

The challenger holds the private key for the one honest server and the two honest clients.

- The adversary picks a plaintext message  $m$  and sends it to the challenger.
- The challenger picks a value  $\beta \in \{1, 2\}$  at random. The challenger sets the slot owner to be honest client  $\beta$ .
- The challenger and adversary run the anonymous communication protocol, with the challenger playing the role of the honest participants and the adversary playing the role of the dishonest participants.

- At the conclusion of the protocol, the adversary makes a guess  $\beta'$  of the value  $\beta$ .

The adversary's advantage  $\epsilon$  in the game is  $|\Pr[\beta = \beta'] - \frac{1}{2}|$ . If  $\epsilon$  is negligible, we say that the protocol maintains *slot owner anonymity*.

Note that this formulation of the anonymity game assumes that all participants' keypairs are generated honestly (i.e., that a dishonest node's public key does not depend on an honest node's public key in a way that could harm the security of the protocol). In practice, we assure that this is true using a proof of knowledge, described in Appendix A.

## C.2 Hashing-Generator Scheme

In the following section, we demonstrate that in the random oracle model [5] and under the decision Diffie-Hellman assumption [6], the hashing-generator ciphertext construction satisfies the security properties of Section 5.1. The security arguments for the ElGamal-style scheme and the pairing-based schemes proceed in a similar fashion, but we focus on the hashing-generator construction because it is the most performant of the three variants.

**Proof Sketch (Completeness, Soundness, Zero-Knowledge)** The client and server ciphertexts are non-interactive zero-knowledge proofs of knowledge, adopted directly from prior work [11, 16, 22, 28]. Our completeness, soundness, and zero-knowledge properties of Section 5.1 follow immediately from the completeness, special soundness, and special honest-verifier zero knowledge properties of the underlying proof system.

**Proof Sketch (Integrity)** Since the ciphertext construction maintains soundness (as argued above), any client or server ciphertext that an honest node finds to be valid will have the correct form. Taking the product of  $N$  correctly constructed client ciphertexts and  $M$  correctly constructed server ciphertexts will result in the cancellation of each pair of client/server shared secrets  $g_\ell^{r_{ij}}$  in the product. (This is because both  $g_\ell^{r_{ij}}$  and its inverse are included in the product.) The resulting product, then, will be the slot owner's plaintext message  $m$ .

**Proof Sketch (Anonymity)** An adversary who wants to break the protocol's anonymity has two choices: (1) the adversary can deviate from the protocol specification in some malicious way, or (2) the adversary can follow the protocol specification exactly and try to break the anonymity by passively observing messages from honest nodes.

Consider the adversary's first option (to violate the protocol in a malicious way). The only opportunities that an adversarial client has to deviate from the protocol are to (a) publish an incorrect commitment to its

shared secret or (b) submit an invalid client ciphertext. Option (a) will not help a malicious client, since the invalid commitment will be detected by honest nodes and will halt the protocol. Option (b) will not help a malicious client either, because the soundness property of the zero-knowledge proof system means that an invalid ciphertext will be rejected by the honest server (thereby halting the protocol run).

An adversarial server can also violate the protocol, but these violations will not help it win the anonymity game either. The server can publish an invalid commitment in the setup phase, but honest nodes will detect this. The server can try to manipulate the set of client ciphertexts, but all honest nodes will detect this as well. The server can broadcast an invalid server ciphertext, but honest nodes will detect this also.

The accountability property of the messaging protocol, in combination with the soundness property of the zero-knowledge proof construction, makes it infeasible for the adversary to gain any information by violating the protocol. (Of course, the adversarial nodes can try to collude to disrupt the protocol as well, but collusion will not enable the adversary to learn anything that it does not already know.)

The adversary's other possible strategy is to follow the protocol correctly and to try to guess who the slot owner is based on a successful run of the protocol. We demonstrate that this strategy is ineffective as well: if there exists an efficient algorithm  $A$  that has an advantage in the anonymity game, there is another efficient algorithm  $B$  that has an advantage in the decision Diffie-Hellman (DDH) game, contradicts the DDH assumption.

The input to the algorithm  $B$  is a DDH challenge tuple  $\langle \hat{g}, \hat{g}^x, \hat{g}^y, \hat{g}^z \rangle$  and  $B$  must output "yes" if  $z = xy$  and "no" otherwise. (This generator  $\hat{g}$  is the same generator  $\hat{g}$  used in Section 5.4.)

To use algorithm  $A$  as a subroutine,  $B$  must simulate  $A$ 's view of a run of the protocol. The simulation proceeds as follows, with  $B$  simulating the role of the challenger and with  $A$  playing the role of the adversary:

- The simulator decides which honest client  $\beta \in \{1, 2\}$  will serve as the anonymous slot owner. (We describe the case in which  $\beta = 1$ , but if  $\beta = 2$ , the roles of the two honest clients are simply swapped.)
- The simulator selects public keys for all of the clients and servers. In the setup phase, the simulator programs the key-derivation function KDF, modeled as a random oracle, to (virtually) assign the value  $y$  to the secret shared  $r_{1,1}$  between honest client  $h_1$  and honest server  $s_1$ . (We say that  $r_{1,1}$  is "virtually" set to  $y$  because the simulator does not actually know the value  $y$ . The simulator only needs to reveal  $\hat{g}^y$  to  $A$ , so the simulator never needs to know  $y$  itself.) The sim-

ulator assigns random values to all secrets  $r_{ij}$  shared between every other client/server pair.

- The simulator must publish commitments  $R_{ij} = \hat{g}^{r_{ij}}$  to each of the honest participants' shared secrets. The simulator knows all of the values  $r_{ij}$  (except  $r_{1,1}$ ), so it can compute almost all of these commitments directly. The simulator uses  $\hat{g}^y$  from the DDH challenge as the commitment to  $r_{1,1}$ .
- The simulator receives a message  $m$  from  $A$ .
- Recall that in the hashing-generator scheme, all participants use a public hash function  $H$  (again modeled as a random oracle) to select the group generator used in each message transmission round.

The simulator programs  $H$  to output  $\hat{g}^x$  as the generator in the first transmission round. The simulator uses  $\hat{g}^z$  from the DDH challenge tuple to simulate the honest clients' ciphertexts:

$$C_1 = m(\hat{g}^z)(\hat{g}^x)^{\sum_{j=2}^M r_{1,j}}$$

$$C_2 = (\hat{g}^x)^{\sum_{j=1}^M r_{2,j}}$$

Similarly, the simulated server ciphertext is:

$$S_1 = (\hat{g}^z)^{-1}(\hat{g}^x)^{-\sum_{i=2}^N r_{i1}}$$

Recall that both the client and server ciphertexts must carry non-interactive zero-knowledge proofs of correctness. In the random-oracle model, the simulator can efficiently simulate these proofs using standard techniques [5] (i.e., by picking the “challenge” value used in the proof before generating the proof's commitments).

At the conclusion of the simulation, algorithm  $A$  will output a guess  $\beta' \in \{1, 2\}$  that honest client  $h_{\beta'}$  was the slot owner during the protocol run. Algorithm  $B$  outputs “yes” (the challenge tuple is a Diffie-Hellman tuple) if  $\beta' = 1$  and “no” otherwise.

With probability  $1/2$ , (when  $z = xy$ ),  $B$  will correctly simulate the view of the challenger and  $B$  will win the DDH game with probability  $\epsilon$ . With probability  $1/2$ , (when  $z \neq xy$ ),  $B$  will produce ciphertexts  $C_1$  and  $C_2$  that are group elements selected at random by the DDH challenger. In this latter case,  $B$  will have no advantage in the DDH game (since  $A$  cannot possibly have an advantage in the anonymity game).

Thus, if  $A$ 's advantage in winning the anonymity game is some non-negligible value  $\epsilon$ , then  $B$ 's advantage in winning the DDH game is  $\epsilon/2$  (which is non-negligible).

## D Zero-Knowledge Proof Instantiation

The cryptographic constructions presented in Section 5 make extensive use of non-interactive zero-knowledge

proofs of knowledge. This section presents an example instantiation of one such proof of knowledge to make the technique concrete. The proof-of-knowledge techniques used in Verdict follow primarily from the work of Camenisch and Stadler [11] which follows in turn from earlier work on proofs of knowledge [16, 22, 44].

Proof-of-knowledge protocols are interactive by nature, but they can be made non-interactive by replacing the role of an interacting verifier with a hash function. This technique, developed by Fiat and Shamir [22], allows for security proofs in the “random-oracle model” [5]. To avoid the unwieldy phrase “non-interactive honest-verifier computationally zero-knowledge proof of knowledge,” we write “proof.”

Our construction uses non-interactive proofs based on Schnorr's proof of knowledge of discrete logarithms [44], proof of equality of discrete logarithms [14], and witness-hiding proofs [16] (which demonstrate that the prover knows at least one out of  $n$  secrets).

We will demonstrate an instantiation of the client ciphertext proof of correctness used in the ElGamal-style construction, introduced in Section 5.2. Proofs for the other ciphertext constructions have almost identical form, so we omit their description herein.

This description assumes that all group elements are elements of a group  $G$  of order  $q$  such that the decision Diffie-Hellman problem [6] is hard in  $G$ . Given an ephemeral public key  $R = g^r$ , a ciphertext element  $C$ , server public keys  $\langle B_1, \dots, B_M \rangle$ , and the slot owner's pseudonym public key  $Y$ , the client executes a proof of knowledge over the secrets  $r$  (the ephemeral secret key) and  $y$  (the slot owner's pseudonym secret key). If the client is the slot owner, the client will know the pseudonym secret key  $y$ . If the client is *not* the slot owner, the client will know the ephemeral secret key  $r$ . The proof has the form:

$$\text{PoK}\{r, y : (R = g^r \wedge C = (\prod_{j=1}^M B_j)^r) \vee Y = g^y\}$$

To simplify the notation, we relabel the variables such that each discrete logarithm relationship has the form  $y_i = g_i^{x_i}$ . Note that the  $g$  and  $y$  values are public, while the  $x$  values are known only to the prover (in this case, prover is the client generating the ciphertext).

$$\begin{array}{lll} g_1 = g & x_1 = r & y_1 = R \\ g_2 = \prod_{j=1}^M B_j & x_2 = r & y_2 = C \\ g_3 = g & x_3 = y & y_3 = Y \end{array}$$

To rewrite the proof statement with the new variable names:

$$\text{PoK}\{x_1, x_2, x_3 : (y_1 = g_1^{x_1} \wedge y_2 = g_2^{x_2} \wedge x_1 = x_2) \vee y_3 = g_3^{x_3}\}$$

The proof has three phases: commitment, challenge, and response. Application of the Fiat-Shamir heuris-

tic [22] means that interaction with the verifier in the “challenge” phase is replaced by a call to a hash function  $\mathcal{H}$  (modeled as a random oracle).

There are two versions of this proof: one in which the prover knows  $x_1$  and  $x_2$  (recall that  $x_1 = x_2$ ), and another in which the prover knows  $x_3$ . These proofs have similar structure, so we present only the former variant.

**Commitment** The commitment values  $(t_1, t_2, t_3)$  are:

$$v_1, v_2, w \in_R \mathbb{Z}_q$$

$$t_1 = g_1^{v_1} \quad t_2 = g_2^{v_1} \quad t_3 = y_3^w g_3^{v_2}$$

**Challenges** The challenge values  $(c_1, c_2)$  are:

$$h = \mathcal{H}(g_1, g_2, g_3, y_1, y_2, y_3, t_1, t_2, t_3)$$

$$c_1 = h - w \pmod q$$

$$c_2 = w$$

**Response** The response values  $(r_1, r_2)$  are:

$$r_1 = v_1 - c_1 x_1 \pmod q$$

$$r_2 = v_2$$

The final proof is  $(c_1, c_2, r_1, r_2)$ .

**Verification** To verify the proof, the verifier first recreates the commitments:

$$t'_1 = y_1^{c_1} g_1^{r_1} \quad t'_2 = y_2^{c_1} g_2^{r_1} \quad t'_3 = y_3^{c_2} g_3^{r_2}$$

If the proof is valid, then the  $t'$ s values should be equal to the original  $t$  values (recall that  $x_1 = x_2$ ):

$$\begin{aligned} t'_1 &= y_1^{c_1} g_1^{r_1} & t'_2 &= y_2^{c_1} g_2^{r_1} \\ &= y_1^{h-w} g_1^{v_1 - c_1 x_1} & &= y_2^{h-w} g_2^{v_1 - c_1 x_1} \\ &= (g_1^{x_1})^{h-w} g_1^{v_1 - c_1 x_1} & &= (g_2^{x_2})^{h-w} g_2^{v_1 - c_1 x_2} \\ &= g_1^{hx_1 - wx_1 + v_1 - (h-w)x_1} & &= g_2^{hx_2 - wx_2 + v_1 - (h-w)x_2} \\ &= g_1^{hx_1 - wx_1 + v_1 - hx_1 + wx_1} & &= g_2^{hx_2 - wx_2 + v_1 - hx_2 + wx_2} \\ &= g_1^{v_1} & &= g_2^{v_1} \\ &= t_1 & &= t_2 \end{aligned}$$

$$\begin{aligned} t'_3 &= y_3^{c_2} g_3^{r_2} \\ &= y_3^w g_3^{v_2} \\ &= t_3 \end{aligned}$$

Finally, the verifier confirms that  $c_1 + c_2 \stackrel{?}{=} \mathcal{H}(g_1, g_2, g_3, y_1, y_2, y_3, t_1, t_2, t_3)$ .

**Security** The protocol described above satisfies standard security properties for proof-of-knowledge protocols [16]. We briefly summarize these properties and sketch a proof that they hold for this proof-of-knowledge.

- **Completeness:** an honest verifier always accepts a proof generated by an honest prover. This is verified by simply observing (by algebraic manipulation) that any valid proof generated by the prover will satisfy the verification equations.

- **Special soundness:** given any valid pair of commitment-challenge-response transcripts  $(t_1, t_2, t_3, c_1, c_2, r_1, r_2)$  and  $(t_1, t_2, t_3, c'_1, c'_2, r'_1, r'_2)$  such that  $c_1 \neq c'_1$  and  $c_2 \neq c'_2$ , it is possible to extract the prover’s secret value used to generate the proof of knowledge.

Consider two accepting conversations generated using the secret value  $x_1$ :

$$(t_1, t_2, t_3, c_1, c_2, r_1, r_2)$$

$$(t_1, t_2, t_3, c'_1, c'_2, r'_1, r'_2)$$

If both proofs are valid, then these relations holds (by the verification equation):

$$r_1 = v_1 - c_1 x_1 \quad r'_1 = v_1 - c'_1 x_1$$

Recovering the secret value  $x_1$  (and hence demonstrating special soundness) requires solving this system for  $x_1$ . The equations are independent since  $c_1 \neq c'_1$  by definition.

$$x_1 = \frac{r_1 - r'_1}{c'_1 - c_1}$$

- **Honest-verifier computational zero-knowledge:** there is a simulator that, when given the statement and the challenge  $h$  as input, produces transcripts computationally indistinguishable from those that an honest prover would produce in responding to  $h$ .

It is straightforward to construct a simulator that, when given  $h$ , produces accepting transcripts that are computationally indistinguishable from a prover-generated transcript.

The simulator receives  $h$  as input and picks a random exponent  $w \in_R \mathbb{Z}_q$ . The simulator sets:

$$c_1 = h - w$$

$$c_2 = w$$

The simulator then picks  $v_1, v_2 \in_R \mathbb{Z}_q$ , and sets the responses:

$$r_1 = v_1 \quad r_2 = v_2$$

The commitments are then:

$$t_1 = y_1^{c_1} g_1^{v_1} \quad t_2 = y_2^{c_1} g_2^{v_1} \quad t_3 = y_3^{c_2} g_3^{v_2}$$

Simple substitution into the verification equation confirms that the simulator’s transcript is valid and that it is computationally indistinguishable from a prover-generated one.

## E Lazy Proof Verification

The *lazy proof verification* technique described in Section 5.4 is an optimization in which servers to process client ciphertexts without checking the ciphertexts’ proofs of validity. Servers only check the client proofs

if they fail to recover the slot owner’s plaintext message later in the protocol run.

Implementing lazy proof verification is straightforward: when the slot owner submits the plaintext message  $m$  to a run of the protocol, the slot owner constructs  $m$  such that it consists of the plaintext message  $m'$  and a cryptographic signature (using the slot owner’s pseudonym signing key) of  $m'$ :<sup>5</sup>

$$m = \langle m', \text{SIGN}_{\text{SlotOwner}}(m') \rangle$$

The slot owner then submits  $m$  as the plaintext message to the Verdict protocol run. When servers reach the point in the protocol in which they obtain the slot owner’s plaintext message  $m$ , they parse  $m$  into  $\langle m', \sigma \rangle$ . If  $\sigma$  is a valid signature on  $m'$ , the servers return  $m'$  to the clients as the slot owner’s plaintext. Otherwise, the servers check each of the client proofs to expose the dishonest client.

Lazy proof verification introduces a subtle security issue that makes it insecure with the ElGamal-style construction (but secure with the other schemes). If servers use lazy proof verification with the ElGamal-style ciphertext construction, a malicious client could submit a ciphertext that “cancels out” the ciphertexts of other clients, thereby violating the security properties of the system.

For example, consider a protocol run in which there are three clients—two honest clients and one dishonest client—and two honest servers with public keys  $g^a$  and  $g^b$ . Using the ElGamal-style ciphertext construction, The two honest clients might submit the following ciphertext tuples:

$$\begin{aligned} \langle R_1, C_1, \text{PoK}_1 \rangle &= \langle g^r, mg^{(a+b)r}, \text{PoK}_1 \rangle \\ \langle R_2, C_2, \text{PoK}_2 \rangle &= \langle g^s, g^{(a+b)s}, \text{PoK}_2 \rangle \end{aligned}$$

Given these two correct client ciphertexts, the dishonest client makes a guess that client 1 is the slot owner. To confirm this guess, the dishonest client constructs a client ciphertext with an *invalid* proof of knowledge:

$$\begin{aligned} \langle R_3, C_3, \text{PoK}_3 \rangle &= \langle R_2^{-1}, C_2^{-1}, \text{PoK}_3 \rangle \\ &= \langle g^{-s}, g^{-(a+b)s}, \text{PoK}_3 \rangle \end{aligned}$$

The dishonest client can construct this ciphertext because it is easy to invert elements of the groups we use. The proof of knowledge  $\text{PoK}_3$  is invalid because dishonest client 3 does not know the secret exponent  $-s$  and therefore cannot (by the special soundness property) cannot construct a valid proof of knowledge.

If the servers accept these three ciphertexts without verifying the accompanying proofs of knowledge, the

<sup>5</sup>In practice, the value  $m$  is encoded as a *vector* of group elements. To simplify our exposition, we pretend in this section that  $m$  consists of a single group element.

servers will generate their server ciphertext elements in response to the ciphertexts  $\langle C_1, C_2, C_3 \rangle$ :

$$S_1 = g^{-a(r+s-s)} \quad S_2 = g^{-b(r+s-s)}$$

The servers then compute the slot owner’s plaintext message  $m^*$  as:

$$\begin{aligned} m^* &= (C_1 C_2 C_3)(S_1 S_2) \\ &= (mg^{(a+b)r} g^{(a+b)s} g^{-(a+b)s}) (g^{-a(r+s-s)} g^{-b(r+s-s)}) \\ &= (mg^{(a+b)r} g^{(a+b)s} g^{-(a+b)s}) (g^{-ar} g^{-br}) \\ &= mg^{ar+br} g^{as+bs} g^{-as-bs} g^{-ar-br} \\ &= m \\ &= \langle m', \text{SIGN}_{\text{SlotOwner}}(m') \rangle \end{aligned}$$

Since the servers recover the slot owner’s original signed plaintext message  $m'$ , the servers will not suspect any malicious behavior on the part of the dishonest client.

The dishonest client, however, has learned that client 1 is the anonymous slot owner. The dishonest client knows this because his ciphertext message  $C_3$  effectively cancels out the effect of honest client  $C_2$ ’s ciphertext (because  $C_3$  is the inverse of  $C_2$ ). If instead client 2 was the anonymous slot owner, then the client 3’s malicious ciphertext would have cancelled out the plaintext message encoded in client 2’s ciphertext, and the servers would have concluded that  $m^* = 1$ . Since the servers successfully recovered the slot owner’s plaintext message, client 3 correctly concludes that client 1 was the slot owner.

The sketch above demonstrates why lazy proof verification is insecure when used with ElGamal-style ciphertexts. We now argue that lazy proof verification is secure in the hashing-generator construction. (The security argument for the pairing-based scheme has a similar structure.)

In the hashing-generator construction, every client  $i$  and every server  $j$  share a secret  $r_{ij}$ . Consider the attack scenario above, with two honest clients and one dishonest client. The honest clients’ ciphertexts in messaging round  $\ell$  will be:

$$\begin{aligned} C_1 &= mg_\ell^{r_{11}+r_{12}} \\ C_2 &= g_\ell^{r_{21}+r_{22}} \end{aligned}$$

The dishonest client can try a trick similar to the one demonstrated before—she constructs a ciphertext that contains her own ciphertext multiplied by the inverse of  $C_2$ :

$$\begin{aligned} C_3 &= C_3' C_2^{-1} \\ &= g_\ell^{r_{31}+r_{32}} g_\ell^{-r_{21}-r_{22}} \end{aligned}$$

The servers then construct their ciphertexts:

$$\begin{aligned} S_1 &= g_\ell^{-r_{11}-r_{21}-r_{31}} \\ S_2 &= g_\ell^{-r_{12}-r_{22}-r_{32}} \end{aligned}$$

When the servers take the product of all of the client and server ciphertexts to reveal the slot owner's message  $m^*$ , the result is corrupted:

$$\begin{aligned}
m^* &= (C_1 C_2 C_3)(S_1 S_2) \\
&= (m g_\ell^{r_{11}+r_{12}} g_\ell^{r_{21}+r_{22}} g_\ell^{r_{31}+r_{32}-r_{21}-r_{22}}) \\
&\quad (g_\ell^{-r_{11}-r_{21}-r_{31}} g_\ell^{-r_{12}-r_{22}-r_{32}}) \\
&= m g_\ell^{r_{11}+r_{12}} g_\ell^{-r_{11}-r_{21}-r_{12}-r_{22}} \\
&= m g_\ell^{-r_{21}-r_{22}}
\end{aligned}$$

In this case, the servers will notice the corruption, will verify the client proofs of knowledge, and will expose client 3 as dishonest. In addition, since the slot owner's plaintext message would also be corrupted if the dishonest client chose to attack client 1's ciphertext, the dishonest client does not learn which honest client is the slot owner.

This attack fails in the general case as well. In the hashing-generator construction, each honest server produces the same server ciphertext element in messaging round  $\ell$  irrespective of the ciphertexts that the clients submit in that round. (This is in contrast with the ElGamal-style construction, in which server ciphertexts depend on the client ciphertexts.) A dishonest client therefore does not learn *any* additional information from an honest client or server by submitting an incorrectly formed ciphertext. Lazy proof verification is then secure when using the hashing-generator ciphertext construction.